The Paradox of Statistical Collapse in Goldbach's Conjecture: An Approach Based on the Laplace Distribution and the Dirac Delta Function

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Abstract

This paper presents a probabilistic framework for analyzing Goldbach's strong conjecture, inspired by the statistical modeling proposed by Maximiliano Mozetic (2025). The central argument revolves around the so-called *paradox of statistical collapse*: when modeling the frequency of Goldbach partitions through a Laplace distribution centered at $N/2$, the distribution converges, in the asymptotic limit $N \to \infty$, to a Dirac delta function. This collapse concentrates all probability at the point $N/2$, which leads to a structural contradiction, since for most even integers N, the value $N/2$ is composite. The implications of this result are discussed in relation to the validity of the conjecture and the limitations of statistical heuristics compared to the rigor of analytic number theory.

1. Introduction

Goldbach's strong conjecture, formulated in 1742, asserts that every even integer $N > 2$ can be expressed as the sum of two prime numbers:

$$N = p_1 + p_2, p_1, p_2 \in \mathbb{P}.$$

Despite extensive computational verification (up to $N \approx 4 \times 10^{18}$) and centuries of theoretical effort, the conjecture remains unproven.

The approach discussed here introduces a probabilistic framework inspired by statistical distribution theory, aiming to explore the structural limitations of the conjecture in the asymptotic regime. The core of the argument is the *paradox of statistical collapse*, formalized in the following sections.

2. Generating Functions and the Laplace Distribution

Let $N \in 2\mathbb{Z}$ be an even integer. Define the frequency function $f_N(x)$ that counts the number of pairs (p_1, p_2) such that $p_1 + p_2 = N$ and $p_1 = x$.

Mozetic proposes approximating $f_N(x)$ by a Laplace distribution:

$$f_N(x) \approx \frac{1}{2b} \exp\left(-\frac{|x - N/2|}{b}\right),$$

where $b > 0$is a scale parameter depending on N.

This model reflects the symmetry of Goldbach partitions around $N/2$, suggesting that the highest density of prime pairs is concentrated near this midpoint.

3. The Paradox of Statistical Collapse

The critical step arises when considering the asymptotic behavior of the scale parameter b.

- As $N \to \infty$, it is postulated that $b \to 0$.
- Consequently, the Laplace distribution converges to a Dirac delta function:

$$\lim_{b \to 0} f_N(x) = \delta(x - N/2).$$

This implies that all probability mass collapses to the single point $x = N/2$.

The paradox can be stated as follows:

- In the limit, the only possible partition is $(N/2, N/2)$.
- For this partition to be valid, $N/2$must be prime.
- However, for almost all $N > 4$, $N/2$is composite.

Thus, the model suggests that in the asymptotic limit, no prime pairs exist that sum to N, contradicting Goldbach's conjecture.

4. Implications and Discussion

The consequences of this reasoning are significant:

- **Potential refutation of the conjecture:** If the probabilistic model could be formalized rigorously, it would imply the existence of a counterexample at some sufficiently large even integer.
- **Limits of statistical intuition:** The result illustrates how massive empirical verification and probabilistic heuristics may fail when extrapolated to infinity.
- **Contrast with classical models:** The Hardy–Littlewood heuristic predicts the abundance of prime pairs for all N, in direct opposition to the singular collapse described by the Laplace model.

This approach does not constitute a formal proof but rather an epistemological critique, highlighting tensions between statistical intuition and arithmetic structure.

5. Conclusion

The paradox of statistical collapse reveals a conceptual boundary in the use of probabilistic models to address problems in number theory. While it does not directly invalidate Goldbach's conjecture, it demonstrates that certain statistical approaches may self-refute in the asymptotic limit.

Future research should explore variants of the model—such as bimodal distributions or Bayesian frameworks—that preserve the duality of prime pairs without collapsing into destructive uniqueness.